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Research of impact input rate random variations on macroscopic characteristics of non-stationary queuing system

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Abstract. The results of research of input stream rate random variations influence on the macroscopic characteristics of a nonstationary queuing system (NQS) are discussed in this paper. Statistical information obtained during the football match between the football clubs "Krylia Sovetov" and "Dynamo" at the stadium "Metallurg" in Samara is available. The characteristics of the stochastic component of the variation in the input rate were chosen on the basis of this information. The probability density function and the cumulative distribution function of random variations in the input rate of applications were estimated using the Rosenblatt-Parzen approximation. NQ's characteristics was studied such as the maximum queue length, the maximum waiting time in the queue, the time point at which the maximum queue length is reached, and the time point at which waiting time in the queue the longest, the number of visitors who entered at the time of the match beginning, the time needed to service the entire queue. The results of the statistical simulation shown that taking into account the random component of the variation input rate does not affect this macroscopic characteristics.

1. Introduction

Nowadays, nonstationary queuing systems (NQS) are widely used, for example, as physical access control systems (PACS) providing access to objects of mass events, devices for passenger check at airports [1] and railway stations, etc. The study of the features of the functioning of such systems is of great interest from a practical point of view. The obtained results could be used as a scientific justification for the design decisions that are accepted at the design and modernization stage of NQS. The difference between NQS and classical queuing systems (QS) is that the input rate of requests of NQS λ and the rate of their service μ can vary with time[2], including randomly.

Analytical study of such systems can be carried out for a small number of NQS types [3]. So it is expedient to examine such queuing systems by simulation models. The using of this method of investigating quantitative characteristics has been demonstrated by the authors in [4]. A piecewise constant approximation of the function $\lambda=\lambda(t)$ is used in this approach:

$$\lambda(t)=\sum_{k=0}^{K}\left(\theta\left(t-t_k\right)-\theta\left(t-t_{k+1}\right)\right)\cdot\bar{\lambda}_k, \tag{1}$$

where

$\theta(t-\xi)$ is Heaviside step function:

$$\theta(t-\xi)=\begin{cases}0, & t<\xi,\\ 1, & t\geq\xi,\end{cases} \tag{2}$$

$k = \overline{1, K}$, K is the number of intervals of piecewise linear approximation $\lambda_{exp}(t)$ and $\bar{\lambda}_k$ is average value of the function $\lambda_{exp}(t)$ on the interval $[t_k, t_{k+1}]$.

It is assumed that at each of the intervals $[\tau_k, \tau_{k+1}]$ the simulated system can be considered as the QS input of which receives the stationary Poisson flow of requests with rate λ_k, and its initial state corresponds to the finite state of the QS on the interval $[\tau_{k-2}, \tau_{k-1}]$. The results of the analysis of information on the dynamics of the entrance to the football stadium gathered during the football matches show, that the rate of the requests flow for NQS input is an additive mix of deterministic and random components:

$$\bar{\lambda}_k = \bar{\lambda}_k^{\det} + \bar{\lambda}_k^{rnd} \quad (3)$$

The article deals with the case when the statistical properties of the random component of the input rate are equals to the statistical properties of the actually observed dependences $\bar{\lambda}_k^{rnd}$. Research is based on the statistical modeling of the NQS. In section 2 present the NQS model (the input flow and the service rate model). In section 3 types, parameters of the investigated input rate, and studied quantitative characteristics of the NQS are described. In section 4 research results of the influence of the NQS's input rate stochastic component on the estimates of studied quantitative characteristics. The main purpose of this article is to study indicators like maximum queue length or the stadium filling at the beginning of the match in different stochastic components condition of the input rate.

2. Mathematical model of the nonstationary queue system

The block diagram of a model of a nonstationary single-channel QS with an unlimited queue is shown on the figure 1. The figure 1 shows that the input of the QS model receives a flow of requests with the rate $\lambda = \lambda(t)$, which varies in time. The main characteristic of the input events flow is the instantaneous flux density $\lambda(t)$. That characteristic is calculating by the formula (4):

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\mathrm{E}(t+\Delta t) + \mathrm{E}(t)}{\Delta t} = \frac{d\mathrm{E}(t)}{dt} \quad (4)$$

where $\mathrm{E}(t)$ is an expected value of the number of events on the interval $[0, t]$.

The FIFO (First In, First Out) policy is using to service the visitor queue. The service speed of incoming requests is determined by the service rate $\bar{\mu}$, which can be characterized by a service time ξ. ξ is a random variable with a probability density function $p\{\xi\}$ (5).

$$p\{\xi\} = \begin{cases} 0, \text{ when } \xi < 1, \\ \dfrac{2}{9(\mathrm{E}[\xi]-1)}(\xi - 1), \text{ when } 1 \le \xi < \mathrm{E}[\xi], \\ \dfrac{2}{9(\mathrm{E}[\xi]-10)}(\xi - 10), \text{ when } \mathrm{E}[\xi] < \xi \le 10, \\ 0, \text{ when } \xi > 10, \end{cases} \quad (5)$$

where $\xi \in [1, 10]$. In the research we used random numbers which were generating in accordance with the probability density (5) and $\mathrm{E}[\xi] = 4$ seconds, $\bar{\mu} = 15$ people per minute.

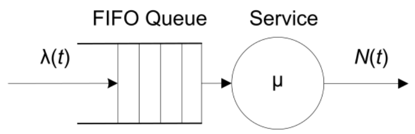

Figure 1. The block diagram of non-stationary QS.

The figure 2.a shows a typical dependency $\lambda_{exp}(t)$, which was obtained during a football match between football clubs "Krylia Sovetov" and "Dynamo" at the stadium "Metallurg" in Samara on 05.05.2013 [5].

The figure 2.a allows us to conclude that the stadium was opened for 1.5 hours before the start of the football match. The input rate of requests increased from 0 person per minute to $\lambda_{max} = 28$ person per minute for time $T_1 \approx 70$ minutes, after turnstiles were opened. The input rate of requests began to decrease approximately 15 minutes before the start of the match. Over the next 50 minutes $T_2 \approx 50$), the number of requests decreased from $\lambda_{max} = 28$ person per minute to 0 person per minute. The total number (6) of visitors who entered through one access control device (turnstile) to the stadium "Metallurg" was 1400 people:

$$N = \int_{-80}^{40} \lambda(t)\,dt. \tag{6}$$

The figure 2.a shows that the dependence $\lambda_{exp}(t)$ is an additive mix of deterministic and stochastic component:

$$\lambda_{\exp}(t) = \lambda_{\det}(t) + \lambda_{rnd}(t). \tag{7}$$

The search for a successful approximation of the deterministic component revealed that the dependence $\lambda(t)$ has a pronounced maximum at the point $t = T_0$. That is why a piecewise polynomial approximation $\lambda_{apr}(t)$ dependence had been used to select the random component $\lambda_{rnd}(t)$ from the experimental dependence $\lambda_{\exp}(t)$ on the intervals T_1, T_2 (figure 2.a).

The figure 2.b shows that the residuals of piecewise polynomial approximation (dependence $\lambda_{rnd}(t)$) are random numbers. Approximation of the Rosenblatt-Parsen [6][7] with a Cauchy kernel function was used to estimate their probability density function and the cumulative distribution function, because it provides the lowest value of the information functional (figure 3).

In the conducted research, the obtained evaluation of the distribution function was used to generate a random component of the input rate of the NQS's requests. The deterministic component of the rate of of arrivals to NQS input is described by the function (8):

$$\lambda_{\det}(t) = \sum_{k=0}^{K} (\theta(t - t_k) - \theta(t - t_{k+1})) \cdot \bar{\lambda}_k^{\det}, \tag{8}$$

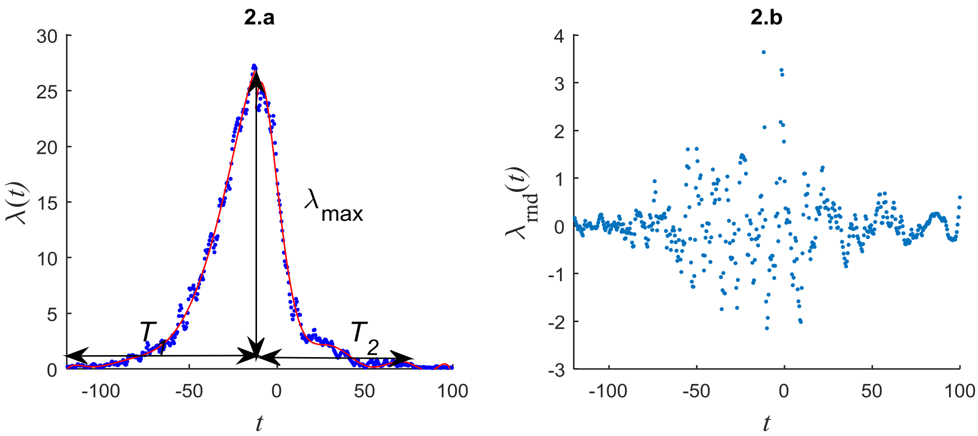

Figure 2. a. Visualization of curve $\lambda_{apr}(t)$ approximating the dependence $\lambda_{\exp}(t)$. b. Residuals $\lambda_{rnd}(t)$ of piecewise polynomial approximation of input rate.

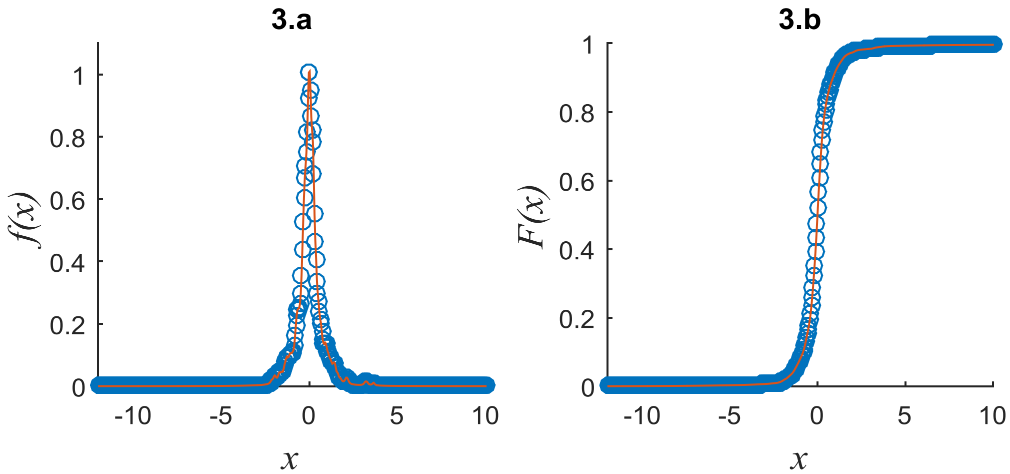

Figure 3. a. Rosenblatt-Parzen approximation of the probability density function of the input rate random component $\lambda_{rnd}(t)$; b. The cumulative distribution function of input rate random component $\lambda_{rnd}(t)$.

where $\theta(t–\xi)$ is Heaviside step function by (2) and $\overline{\lambda}_k^{\det}$ is mean value of the function $\lambda(t)$ on the interval $[t_k, t_{k+1}]$.

The number of intervals K of piecewise constant approximation of the dependence $\overline{\lambda}_k^{\det}$, was chosen equal to 1520. This value is based on previous researches [8].

3. Computational experiments methods

The following kinds of dependences of rate of an input flow of requests from time were used at carrying out of computational experiments:

- $\lambda_{\det}(t)$ is function whose value is calculated according to (8);
- the sum of the values of the $\lambda_{\det}(t)$ function computed in accordance with (8) and the random sequence generated in accordance with the cumulative distribution function estimated from the experimental data;
- the sum of the values of the $\lambda_{\det}(t)$ function computed in accordance with (8) and the random sequence generated in accordance with the cumulative distribution function estimated from the experimental data at each stage of statistical modeling.

Figure 4 illustrates examples of the dependencies of the input rate of the flow of requests types 2 and 3.

The arrival times of requests t^A were generated during each interval $[t_i;\ t_{i+1}]$ using constant approximation of the input rate. These time values correspond to the exponential distribution law with rate equal to the average value of the arrival rate at the interval. The service time interval τ^S was generated in accordance with (5). Time of entering in service t^E for each request in queue calculated next. That is why all requests that stood during this interval time in the service queue were alternately looked through:

$$t_i^E = \begin{cases} t_i^A,\ t_i^A \ge t_{i-1}^E + \tau_{i-1}^S, \\ t_i^A + (t_{i-1}^E + \tau_{i-1}^S - t_i^A),\ t_i^A < t_{i-1}^E + \tau_{i-1}^S. \end{cases} \tag{9}$$

It is obvious that for the first request $t_1^E = t_1^A$.

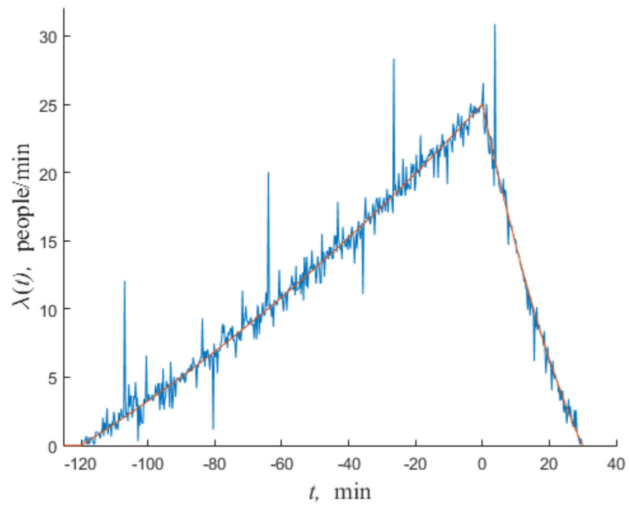

Figure 4. $\lambda(t)$ are dependences of non-stationary QS (the beginning of the match $t = 0$).

Let's denote the set of requests as Q, and q_n like the element of this set. We used the dependence of the queue length of visitors (the requests queue length in terms of QS) from time for quantitative describe the features of the NQS data operation according to [5]:

$$L = L(t_k) = |Q|, \text{ when } Q = \{q_n : t_n^A < t_k \cap t_n^E > t_k\}, \tag{10}$$

Also average visitor waiting time (request waiting time in terms of QS) in the queue from time is useful:

$$\tau^w = \tau^w(t_k) = \frac{\sum_{i=1}^{|Q|}(t_i^E - t_i^A)}{|Q|}, \text{ when } Q = \{q_n : t_n^E \le t_k \cap t_n^E > t_{k-1}\}, \tag{11}$$

Dependence of the number of visitors entering the stadium from time (the number of serviced requests in terms of QS):

$$N_{out} = N_{out}(t_k) = |Q|, \text{где } Q = \{q_n : t_n^E < t_k\}, \tag{12}$$

where $t_k = T_1 + \frac{T_2 - T_1}{K}(k-1)$.

4. Analysis of experimental results

Examples of the dependencies of the chosen quantitative characteristics are shown in figure 5.

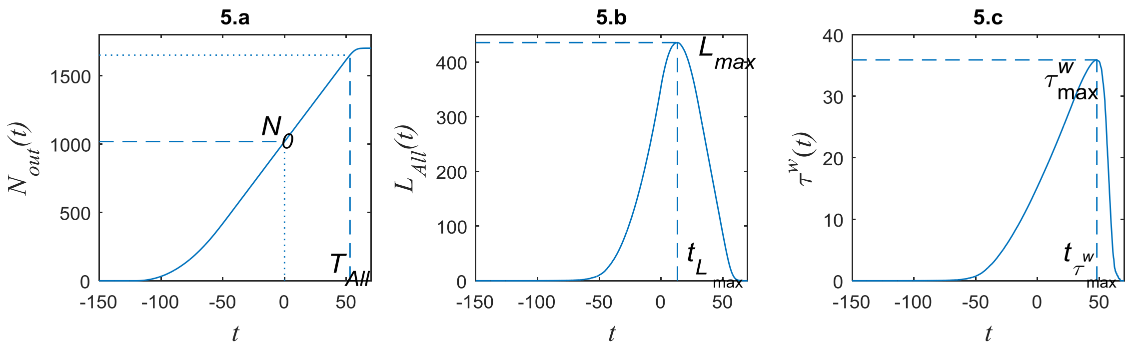

Figure 5. Dependencies: a. the numbers of entered from time $N_{out}(t)$ b. the length of the queue from the time $L(t)$ c. waiting time for maintenance from time $\tau^w(t)$.

According to the graphs, the averaged dependencies of the NQS characteristics on time should be considered as functions. These functions have significant values that can be useful for engineers, designers and also appropriate security services. We propose to call them *macroscopic* characteristics of NQS. Figure 5 shows that the dependencies of the queue length and waiting time in the queue from time can be characterized by the following macroscopic features:

- The maximum value $x_{\max}$

$$x_{\max} = \max(x), \text{ when } x \in \{L, \tau^w\}. \tag{13}$$

- The studied dependence reaches maximum value with argument value $t_{X\max}$

$$t_{x_{\max}} = \arg\max(x), \text{ when } x \in \{L, \tau^w\}. \tag{14}$$

Another macroscopic quantitative characteristic is the number of visitors who entered at the time of the beginning of match:

$$N_0 = |Q|, \text{ when } Q = \{q_n : t_n^E < t = 0\}, \tag{15}$$

The time required to service all incoming visitors is macroscopic quantitative characteristic too:

$$T_{All} = \{t : N(t) \geq 0.97 \cdot N_{\max}\}, \text{ when } N_{\max} = \max(N(t)). \tag{16}$$

Random sequences are set of values that calculated at each step of the Monte Carlo method. The samples obtained are poorly described by continuous distribution laws. In connection with this, nonparametric statistics are used. Their distribution density function was approximated by Rosenblatt-Parzen method [6][7]. Estimates of the quantiles of the studied random sequences are presented in table 1. Table 1 shows that the values of the corresponding quantiles of the distributions of the selected macroscopic characteristics of NQS turned out to be different from each other. The statistical significance of data differences check was in accordance with the Kolmogorov-Smirnov tests. This is a two-sided test for the null hypothesis that 2 independent samples are drawn from the same continuous distribution. It turned out that the differences in the studied distributions of the macroscopic characteristics of NQS for each of the methods of specifying the input flow of requests rate at the level of confidence probability $p=10^{-3}$ are statistically insignificant. As can be seen from table 2, for all macroscopic characteristics of NQS for all 3 studied input intensities, the evaluation of statistics is close to zero. This means that the null hypothesis that it is two samples of one general population is rejected. Consequently, if input rate random components of studied of NQS do not exceed researched input rate stochastic variations (residuals of piecewise polynomial approximation) then it turns out to be sufficient to use a dependence whose values are calculated in accordance with (8).

Table 1. Estimates of the quantiles of the distributions of quantitative indicators of NQS.

Parameter	$\lambda(t)$ computing method 1			$\lambda(t)$ computing method 2			$\lambda(t)$ computing method 3		
CI	0.05	0.5	0.95	0.05	0.5	0.95	0.05	0.5	0.95
$L_{\max}$,people	371.3	439.4	512.2	369.4	437	508.9	372.7	438.9	502.1
$t_{L_{\max}}$, min	8.8	12.8	16.9	8.3	12.1	16.2	8.2	12.4	16.5
$\tau^w_{\max}$, min	31.0	36.6	42.5	30.6	36.4	42.0	31.0	36.6	41.9
$t_{\tau^w_{\max}}$, min	42.0	49.5	57.5	41.3	48.7	56.1	42.0	49.4	56.8
N_0, people	979.0	1018.6	1057.5	981.3	1019.8	1058.8	979.6	1018.3	1059.2
T_{All}, min	46.8	52.8	59.4	46.1	52.2	58.4	46.8	52.7	58.7

Table 2. Two-samples Kolmogorov-Smirnov statistics results of quantitative indicators of NQS.

Parameter	λ(*t*) 1 and λ(*t*) 2	λ(*t*) 2 and λ(*t*) 3	λ(*t*) 1 and λ(*t*) 3
$L_{\max}$,people	0.072	0.048	0.062
$t_{L_{\max}}$, min	0.054	0.058	0.054
$\tau^{w}_{\max}$, min	0.064	0.042	0.064
$t_{\tau^{w}_{\max}}$, min	0.028	0.040	0.050
N_0 , people	0.042	0.066	0.044
T_{All} , min	0.040	0.026	0.052

5. Conclusions

The estimates of the macroscopic characteristics of the non-stationary queuing system under piecewise-polynomial variation of the deterministic component of the input intensity $\lambda_{\det}(t)$ (8) in the range [0; 25] person per min were investigated. Also, estimates of the macroscopic characteristics of the NQS at the input rate as a mix of a deterministic and a random component (7) are considered. The range [–3; 4] person per min and the distribution density function of the received random component were obtained on the basis of the available statistical information collected during the football match between the football clubs "Krylia Sovetov" and "Dynamo" at the stadium "Metallurg" in Samara. As a result, it can be concluded that the presence of a random component of the input intensity does not have a significant effect on estimating the quantitative characteristics of NQS. Subsequent publications will give the answer which values of $\lambda_{rnd}(t)$ significantly influence on the macroscopic characteristics of NQS. The theme for future work may be the detection of such input rate random variations at which the values of the NQS macroscopic parameters will be statistically different. In further publications, it is required to find the dependence of the macroscopic characteristics of NQS from the signal-to-noise ratio of the input rate.

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